Lotteries and Lindahl prices in public good provision

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Abstract

Lotteries are traditional instruments for fundraising in general. Morgan (2000) has shown that they can also be very effective in the provision of a public good. However, a fair lottery can only enhance provision but never result in the efficient amount. Franke and Leininger (2014) show - by borrowing from optimal contest theory - that *biased* lotteries can provide the efficient amount of the public good. This paper aligns this result with standard public good theory, in particular the classic notion of Lindahl pricing. It shows that biased lotteries can - implicitly - implement Lindahl pricing of the public good in non-cooperative Nash equilibrium.

Key Words: Public good provision, biased lotteries, Lindahl pricing.

JEL classification: C72; D72; H41

1 Introduction

There is a huge literature on the provision of public goods. Morgan (2000), however, was the first to link the old institution of lotteries directly to the theory of the provision of public goods. He views a lottery, part of whose proceeds go to the provision of a public good, as a voluntary contribution scheme (and not merely a substitute for confiscatory tax schemes by the state). And, indeed, many private charities or institutions that lack any taxing power use lotteries to generate revenues for their respective aims.

Lotteries are a worldwide phenomenon (for an extensive study of US state lotteries see e.g. Clotfelter and Coak, 1991) and the traditional view of them as inefficient and regressive instruments for raising surrogate tax money is being challenged by Morgan's question, whether they rather may constitute an effective contribution scheme towards the supply of public goods.

Morgan (2000) showed that the provision of a public good can be enhanced by the use of lotteries or - in Morgan's terms - fixed prize raffles.The reason for this interesting result is that the positive externality of a contribution on others in the pure voluntary provision scheme is now counteracted to some extent by the negative externality on others of buying additional lottery tickets (which lowers their probability of winning the contest for the fixed prize R). As a result contributions towards financing the public good increase in comparison to voluntary contribution. However, any prize sum of finite value is never sufficient to finance and provide the *efficient amount* of the public good. In this regard Morgan's model can be viewed as a fund-raising model as he is not primarily interested in efficient provision of the public good.

Franke and Leininger (2014) directly address efficiency within Morgan's model. By using insights from optimal contest theory (Franke et al., 2013) they show that an appropriately *biased* lottery together with a *finite* prize sum can induce the efficient amount of public good provision, if consumers differ sufficiently in the valuation of the public good. They do so under Morgan's assumption that lottery prizes have to be financed completely out of lottery proceeds. This is in marked contrast to Kolmar and Wagner (2012) and Giebe and Schweinzer (2014) who rely on forms of coercive taxation to finance the lottery prize in order to get an efficiency result. The purpose of this paper is to explain the intuition behind the surprising result of Franke and Leininger (2014) within Morgan's model by analyzing the relationship between optimally biased lotteries and Lindahl prices. The insight from optimal contest theory is that biasing contest success functions - here lotteries - can increase competitive pressure among the contestants - here consumers -, which increases the amount of negative externality exerted on others through ticket purchases. This effect further mitigates the free-riding incentive due to the public good provision. This transmission effect is equivalently represented here as a public good *pricing* mechanism, which directly relates to classic Lindahl pricing of a public good.

The paper is organized as follows. In section 2 we introduce Morgan's (2000) model and review the results obtained for it. Section 3 reinterprets biased raffles as unbiased raffles with individual prices for tickets and shows the main result, that the efficient provision of the public good by means of a biased raffle leads effectively to Lindahl pricing of the public good in (non-cooperative) equilibrium. Section 4 concludes.

2 The Model

There is a set $N = \{1, \ldots, n\}$ of consumers, who each has a quasi-linear utility function of the type

$$u_i(w_i, G) = w_i + h_i(G)$$

with w_i summarizing the wealth of i and G denoting the amount of the public good provided economy wide. It is standard to assume that $h_i^{'} > 0$ and $h_i^{''} < 0$, $i = 1, \ldots, n$. Wealth can be transformed into public good by using the production function $f(w) = w$; i.e. one unit of (private) wealth can be transformed into one unit of the public good. All consumers are (expected) utility maximizers.

A social planner would like to implement the socially optimal amount of the public good (which coincides in this quasi-linear framework with the efficient allocation); i.e. he would choose to provide G^* units of the public good, where G^* maximizes

$$\text{(SO)} \quad \begin{aligned} W &= \textstyle\sum_{i=1}^n u_i(w_i, G) - G \\ &= \textstyle\sum_{i=1}^n (w_i + h_i(G)) - G. \end{aligned}$$

With non-binding wealth constraints the optimal amount G^* has to satisfy the well-known Samuelson condition:

$$\text{(SOC)} \quad \sum_{i=1}^n h_i^{'}(G^*) = 1.$$

If, for simplicity, we also assume that $h_i^{'}(0) > 1$, then the public good is always desirable and $G^* > 0$ should be provided.

The following facts are well-known:

i) *Voluntary contribution schemes provide less than G^* of the public good:* In such a scheme consumers directly contribute an amount x_i, $i = 1, \ldots, n$ to the provision of the public good. Consumer i determines his contribution by maximizing

$$u_i\left(w_i - x_i, \sum_{j=1}^{n} x_j\right) = w_i - x_i + h_i\left(\sum_{j=1}^{n} x_j\right).$$

As Bergstrom et al. (1986) show, this results in $\sum_{j=1}^{n} x_j^* < G^*$ in any Nash equilibrium $x^* = (x_1^*, \ldots, x_n^*)$ of the contribution game.

ii) *Fixed-prize raffles provide less than G^* of the public good:* Morgan (2000) showed that the provision of the public good can be enhanced by the use of special lotteries, which he termed fixed-prize raffles. In such a raffle a pre-announced fixed prize of value R is offered by the prospective provider of the public good; e.g. the government or a charity institution, and awarded to the lucky buyer of the winning lottery ticket. The prize R itself has to be financed out of the proceeds from ticket sales S. So, if ticket sales amount to S, only the amount $S - R$ can be used towards financing the public good. A consumer is hence not asked directly to contribute to the provision of the public good, but indirectly through the purchase of lottery tickets for R (with the remaining proceeds being transformed into G, the public good). Consumer i consequently maximizes

$$Eu_i(x_i, x_{-i}) = w_i - x_i + \frac{x_i}{\sum_{j=1}^{n} x_j} R + h_i\left(\sum_{j=1}^{n} x_j - R\right),$$

where $\frac{x_i}{\sum\limits_{j=1}^{n} x_j}$ denotes the probability of consumer i winning the price R.

This indirect voluntary provision game always has a unique Nash equilibrium (Morgan (2000), Proposition 2). Moreover, the amount of the public good provided in this equilibrium always exceeds the amount provided by the voluntary contribution scheme of case i). Note, that this means that ticket sales not only exceed R, and hence the prize R can be provided, but also that they exceed R *plus* the amount provided by the voluntary contribution scheme. The positive externality in providing amounts of the public good (by privately buying lottery tickets) onto others is now automatically combined with a negative externality onto others as this reduces their chances to win the prize R. Overall the individual incentives to free-ride are sufficiently reduced to provide *more* of the public good *and* - at the same time - retrieve the cost of the prize R (Morgan (2000), Theorem 1). However, even in this situation the efficient amount of the public good cannot be implemented with a finite prize sum; that is, efficient public good provision can only be achieved in the limit for a prize of infinite value which requires consequently unlimited wealth of consumers (Morgan (2000), Theorem 2). This holds also true for more general utility functions than the ones used by Morgan (see Duncan, 2002).

iii) Franke and Leininger (2014) consider a fixed prize raffle with biased lotteries, i.e. each consumer's contribution x_i (via ticket purchases) is weighted by a parameter $\alpha_i > 0$. The winning probability of i is then given by $\frac{\alpha_i x_i}{\sum\limits_{j=1}^{n} \alpha_j x_j}$. Then each consumer i determines the amount of ticket purchases x_i by maximizing the expected payoff

$$\text{(BR)} \qquad u_i(x_1, \ldots, x_n) = w_i - x_i + \frac{\alpha_i x_i}{\sum_{j=1}^{n} \alpha_j x_j} R + h_i\left(\sum_{j=1}^{n} x_j - R\right) \quad i = 1, \ldots, n.$$

A Nash equilibrium $x^* = (x_1^*, ..., x_n^*)$ is given by a vector of ticket purchases such that

$$\begin{aligned} & w_i - x_i^* + \frac{\alpha_i x_i^*}{\sum_{j=1}^{n} \alpha_j x_j^*} R + h_i(\sum_{j=1}^{n} x_j^* - R) \\ \geq \; & w_i - x_i + \frac{\alpha_i x_i}{\alpha_i x_i + \sum_{j \neq i} \alpha_j x_j^*} R + h_i(x_i + \sum_{j \neq i} x_j^* - R) \text{ for all } x_i \in [0, w_i], i = 1, \ldots, n. \end{aligned}$$

Franke and Leininger (2014) demonstrate for the simple case of two consumers that there exist weights α_1 and α_2 and a prize R such that *in equilibrium* of the biased raffle game the efficient amount of the public good is provided. The intuition behind this result is that the heterogeneity of consumers can be balanced to some degree through appropriately specified weights which increases competitive pressure even further up until the efficient amount. Hence, heterogeneity of the consumers is a necessary precondition for biased fixed prize raffles inducing efficient public good provision.

It is the purpose of this paper to align this result with standard microeconomic theory for public goods; in particular the notion of Lindahl equilibrium, which always yields an efficient allocation. In a Lindahl equilibrium each utility maximizing consumer pays a per-unit price for financing the public good equal to his marginal valuation of the good. So everyone may pay a different price, yet total payments exactly suffice to finance the good since all consumers individually demand the efficient amount of the public good. In the present model this means that Lindahl prices $p_L = (p_{1L}, ..., p_{nL})$ have to be such that if a consumer i maximizes

$$u_i\,(w_i - p_{iL} \cdot G, G) = w_i - p_{iL} \cdot G + h_i(G)$$

he should demand G^* units of the public good and total revenue satisfies $\sum_{i=1}^n p_{iL} \cdot G^* = G^*$; i.e. $\sum_{i=1}^n p_{iL} = 1$ must hold. Hence, $p_{iL} = h_i'(G^*), i = 1, ..., n$, has to hold, which automatically yields $\sum_{i=1}^n p_{iL} = \sum_{i=1}^n h_i'(G^*) = 1$ because of (SOC). So Lindahl prices in our model are given by $p_{iL} = h_i'(G^*)$ for $i = 1, ..., n$.

3 Biased raffles and Lindahl prices

We show in detail that a biased raffle which generates winning probabilities according to $Pr_i(x_1, ..., x_n) = \frac{\alpha_i x_i}{\sum_{j=1}^n \alpha_j x_j}$, $i = 1, ..., n$, is equivalent to an *unbiased* raffle with individual ticket prices $p_i = \frac{1}{\alpha_i}$, $i = 1, ..., n$. The latter implies that the raffle organizer can price-discriminate between consumers who buy tickets. To be more precise: the problem (BR) of a consumer in a biased indirect contribution game can be transformed into the following problem *where the equilibrium remains invariant to this transformation*.

Define $y_i := \alpha_i x_i$, $i = 1, \ldots, n$. Then (BR) can be written as

$$\text{(PD1)} \qquad \max_{y_i} w_i - \frac{1}{\alpha_i} y_i + \frac{y_i}{\sum_{j=1}^n y_j} R + h_i\left(\sum_{j=1}^n \frac{1}{\alpha_j} y_j - R\right),$$

and we see that the prize R is now awarded in a fair raffle with ticket price $p_i = \frac{1}{\alpha_i}$ for consumer $i = 1, \ldots, n$. Also note, that revenue from ticket sales now amounts to $\sum_{i=1}^n p_i y_i = \sum_{i=1}^n \frac{1}{\alpha_i} y_i$; i.e. accounting for the cost of the raffle prize leads to a supply of the public good of $\sum_{i=1}^n \frac{1}{\alpha_i} y_i - R$.

The equivalence of the solutions of the biased indirect contribution game defined by (BR) and the unbiased indirect contribution game with price discrimination defined by (PD1) is now immediate: (BR) leads to the first-order conditions

$$(*) \qquad -1 + \frac{\sum_{j \neq i} \alpha_j x_j}{(\sum_{j=1}^n \alpha_j x_j)^2} \cdot \alpha_i \cdot R + h_i'\left(\sum_{j=1}^n x_j - R\right) = 0, \quad i = 1, \ldots, n;$$

while (PD1) results in

$$-\frac{1}{\alpha_i} + \frac{\sum_{j \neq i} y_j}{(\sum_{j=1}^n y_j)^2} R + \frac{1}{\alpha_i} h_i'\left(\sum_{j=1}^n \frac{1}{\alpha_j} y_j - R\right) = 0, \quad i = 1, \ldots, n.$$

If we multiply the latter by α_i, $i = 1, \ldots, n$, they read

$$-1 + \frac{\sum_{j\neq i} y_j}{(\sum_{j=1}^n y_j)^2} \cdot \alpha_i \cdot R + h_i^{'}\left(\sum_{j=1}^n \frac{1}{\alpha_j} y_j - R\right) = 0, \quad i = 1, \ldots, n.$$

So, if $y = (y_1, \ldots, y_n)$ solves the latter equations, then $x = (x_1, \ldots, x_n) = \left(\frac{1}{\alpha_1} y_1, \ldots, \frac{1}{\alpha_n} y_n\right)$ must solve the first system (and vice versa).

Now consider a raffle organizer who can sell tickets at individual prices $p = (p_1, ..., p_n) > 0$. Then consumer i faces the following choice problem:

$$\text{(PD2)} \qquad \max_{z_i} w_i - p_i \cdot z_i + \frac{z_i}{\sum_{j=1}^n z_j} R + h_i\left(\sum_{j=1}^n p_j z_j - R\right).$$

The first-order condition is given by

$$-p_i + \frac{\sum_{j\neq i} z_j}{(\sum_{j=1}^n z_j)^2} R + p_i h_i^{'}\left(\sum_{j=1}^n p_j z_j - R\right) = 0$$

and upon dividing by p_i (> 0) this reads

$$(**) \qquad -1 + \frac{\sum_{j\neq i} z_j}{(\sum_{j=1}^n z_j)^2} \frac{R}{p_i} + h_i^{'}\left(\sum_{j=1}^n p_j z_j - R\right) = 0.$$

From this we see that the effect of imposing a ticket price p_i on player i is equivalent to a revaluation of the prize R to $\frac{R}{p_i}$, which changes incentives of player i. Note, that as long as the provision constraint $\sum_{j=1}^n p_j z_j > R$ is met any additional unit of wealth of player i used for ticket purchases increases the supply of the public good by exactly one unit.

The term $\frac{\sum_{j\neq i} z_j}{(\sum_{j=1}^n z_j)^2} \frac{R}{p_i}$ measures the expected marginal increase in winnings from the raffle, if one additional unit of private wealth is spent on tickets. Hence the *net* payment for the provision of an additional unit of the public good by player i is

$$1 - \frac{\sum_{j\neq i} z_j}{(\sum_{j=1}^n z_j)^2} \frac{R}{p_i} < 1.$$

So personalized ticket prices (resp. personalized prizes R_i) indirectly lead to personalized prices for the public good p_{iG}, $i = 1, ..., n$. We have

$$p_{iG} := 1 - (MW_i)\frac{R}{p_i}$$

where MW_i stands for marginal winnings; in this case $MW_i = \frac{\sum_{j \neq i} z_j}{(\sum_{j=1}^{n} z_j)^2}$. If these (net) prices p_{iG}, $i = 1, ..., n$, would equal Lindahl prices p_{iL}, $i = 1, ..., n$, the raffle should provide the efficient amount of the public good, G^*, and thereby *implement* Lindahl prices in a non-cooperative Nash equilibrium! Alternatively, we claim that if a raffle organizer can find prices $p^* = (p_1^*, ..., p_n^*)$ such that the raffle in equilibrium provides G^*, then it must hold that

$$p_{iG} := 1 - (MW_i)\frac{R}{p_i^*} = p_{iL}.$$

We demonstrate this in our model: suppose that $\sum_{j=1}^{n} p_j z_j - R = G^*$. Then the first-order condition of consumer i's choice problem reads

$$1 - (MW_i)\frac{R}{p_i^*} - h_i'(G^*) = 0, \quad i = 1, ..., n.$$

Recall now that in our simple model with quasilinear utility functions Lindahl prices are given by

$$p_{iL} = h_i'(G^*), \quad i = 1, ..., n,$$

as consumer i exactly demands the amount G^* of the public good in a market setting, where G^* maximizes $u_i(w_i - p_{iL}G, G) = w_i - p_{iL}G + h_i(G)$.
Consequently,

$$p_{iG} = 1 - (MW)\frac{R}{p_i^*} = p_{iL}$$

for all consumers $i = 1, ..., n$, which leads to the following result.

Theorem 3.1 *Suppose an unbiased fixed prize raffle with price-discrimination yields the efficient amount G^* of the public good in equilibrium. Then the public good is effectively priced at Lindahl prices.*

Alternatively, if we consider the biased indirect contribution game with $\alpha_i = \frac{1}{p_i^*}$, $i = 1, ..., n$, it holds that

Corollary 3.2 *The efficient provision of the public good through a biased fixed prize raffle implements Lindahl pricing for the public good.*

Discriminative prices p_i resp. biases $\alpha_i, i = 1, ..., n$, effectively *individualize* the lottery prize R to $\frac{R}{p_i}$ resp. $\alpha_i R$ as can be seen from the first order conditions of (PD1) and PD2 labelled (*) and (**). This instrument - individualized lottery prize(s) - allows the coordination of individual demands for the public good onto a common amount G^*. We have shown that this mechanism implicitly leads to the realization of "Lindahl-shares" in financing the public good, if one controls for the payoffs resulting from positive winning probabilities for these lottery prizes. One common way to implement a kind of price-discrimination is giving discounts to multi-ticket purchases. For more on this see Damianov (2015).

4 An Example

We now provide an example with three heterogeneous players to illustrate our framework and demonstrate our results. We consider the following specific valuation functions in order to solve numerically for the equilibrium: $h_1(G) = G^{1/3}$, $h_2(G) = \frac{1}{2}G^{2/3}$ and $h_3(G) = c\,G^{1/2}$, where $c = 2.631$. Note first, that $h_1(G) > h_2(G)$ if and only if $G < 8$, while $h_1'(G) > h_2'(G)$ if and only if $G < 1$. Hence, the relative strength with respect to valuations (or marginal valuations) of these two players depends on the respective level of public good provision and therefore cannot be determined a-priori.[1] Secondly, for reasons of convenience the constant c in the valuation function of the third player is parametrized such that the optimal amount of the public good is $G^* = 4$.

Private Provision. Private provision of the public good yields individual contributions of $(x_1, x_2, x_3) = (0.192, 0.037, 1.730)$, which leads to inefficient underprovision of the public good: $x_1 + x_2 + x_3 = 1.960 < 4 = G^*$.

Morgan Lottery. Using a fixed prize raffle with value R, the inefficient underprovision result of the private provision model can be ameliorated to some extent: A prize value of $R = 10$, for instance, yields equilibrium investments of $(x_1, x_2, x_3) = (0.335, 1.195, 10.623)$ resulting in $x_1 + x_2 + x_3 - R = 2.153$ of the public good, which is an improvement in comparison to the private provision model.[2] Public good provision can be further increased by choosing higher

[1] In this sense our specification is more involved than the two player example in Franke and Leininger (2014), where depending on the specification of the valuation function either $h_1(G) > h_2(G)$ and $h_1'(G) > h_2'(G)$ for all G or vice versa, as well as the simple contest game analyzed in Franke et al. (2013), where valuations are fixed and players can be easily ranked with respect to their strength (i.e., valuation of the prize).

[2] Morgan (2000) abstracts from wealth constraints in his analysis because he is rather interested in allocative instead of distributive efficiency. Setting higher prize values in the lottery model implies that wealth constraints are more likely to be binding under a given wealth distribution.

prize values: A prize value of $R = 50$ (100) yields an amount of 3.175 (3.498) of the public good. However, as shown in Morgan (2000), no finite prize sum will result in the optimal level of public good provision. In other words, the Morgan lottery framework is not able to neutralize free-riding completely such that inefficient underprovision of the public good still remains a concern.

Biased Lottery. In a biased lottery framework, individual contributions are weighted differently in the process of determining the lottery winner, which opens a further channel to incentivize players to invest into lottery tickets and therefore increase public good provision even further. If players are heterogeneous then the bias can be designed in such a way that the playing field is more leveled. This will increase competitive pressure and induce even higher contributions than under the Morgan lottery. A prize value of $R = 21$ in combination with bias weights $(\alpha_1, \alpha_2, \alpha_3) = (1.66, 1.7, 1)$, for instance, yields equilibrium investments of $(x_1, x_2, x_3) = (4.066, 5.592, 15.339)$, which results in the efficient amount $x_1 + x_2 + x_3 - R = 3.997 \approx G^* = 4$ of the public good. Note, that the incentives in a biased lottery can be so strong that even inefficient overproduction can occur; for instance, with a prize value of $R = 23$ and unchanged bias weights the amount of the public good results in $4.086 > G^*$. Moreover, there might be more than one combination of biases and prize value that results in efficient public good provision; for instance, the combination $(R, \alpha_1, \alpha_2, \alpha_3) = (30, 1.55, 2.38, 1)$ also yields public good provision at the efficient level of $3.999 \approx G^*$.These examples suggest that bias and prize sum can substitute each other to some degree. In the last mentioned specification, for instance, a comparatively higher prize sum is combined with a slightly lower bias weight α_2 (although α_3 has to be increased accordingly to maintain efficiency).

Lindahl Pricing. Using Lindahl prices of $(p_{1L}, p_{2L}, p_{3L}) = (0.132, 0.210, 0.658)$ each player would be willing to demand and finance the efficient amount $G^* = 4$ of the public good and the entire cost of provision is covered because $p_{1L} + p_{2L} + p_{3L} = 1$. Note also, that Lindahl prices help to identify the relative ranking of players at the optimal provision point: Here, the second player benefits more from public good provision at the efficient level than the first player because $p_{2L} = h'_2(G^*) > p_{1L} = h'_1(G^*)$. We can also replicate the established relation between Lindahl prices and optimal weights in the biased lottery framework. From the equality $p_{iL} = 1 - \frac{\sum_{j \neq i} z_j}{\left(\sum_{j=1}^{3} z_j\right)^2} \alpha_i R$ we can recover individual bias weights for the first mentioned specification as follows:

$$\alpha_1 = \frac{(z_1 + z_2 + z_3)^2}{z_2 + z_3} \frac{1 - p_{1L}}{R} = \frac{(6.750 + 9.506 + 15.339)^2}{9.506 + 15.339} \frac{1 - 0.132}{21} = 1.66$$
$$\alpha_2 = \frac{(z_1 + z_2 + z_3)^2}{z_1 + z_3} \frac{1 - p_{2L}}{R} = \frac{(6.750 + 9.506 + 15.339)^2}{6.750 + 15.339} \frac{1 - 0.210}{21} = 1.7$$

$$\alpha_3 = \frac{(z_1 + z_2 + z_3)^2}{z_1 + z_2} \frac{1 - p_{3L}}{R} = \frac{(6.750 + 9.506 + 15.339)^2}{6.750 + 9.506} \frac{1 - 0.658}{21} = 1.$$

5 Discussion

More General Contest Success Functions. An optimally biased fixed prize raffle induces the efficient amount of public good provision by favoring specific players and therefore increasing competitive pressure up to the right amount. We now discuss the robustness of our result with respect to more general contest success function (that could also include alternative ways of favoritism like head starts). Consider, for instance, the following class of contest success functions: $\frac{f_i(x_i)}{\sum_{j=1}^n f_j(x_j)}$, where $f_i(x_i)$ is non-negative, increasing and concave.[3] This class contains the biased lottery or fixed prize raffle as special case ($f_i(x_i) = \alpha_i x_i$), but also other types of favoritism like head starts ($f_i(x_i) = x_i + \delta_i$), where players obtain an up-front lump-sum bonus of lottery tickets. For simple contest games this general class of contest success functions has been analyzed in Franke et al. (2016), section 6.1, where it is shown that within this class an appropriately biased lottery leads to highest total contributions. In other words, competitive pressure under the optimally biased lottery is higher in comparison to any other contest success function of this class. This insight can also be applied to the fixed prize raffle with public good provision, where the objective is to induce the efficient and not the maximal amount of contributions. We therefore conjecture that any other contest success function from this class resulting in efficient public good provision could be substituted by a biased fixed prize raffle with a comparatively lower prize sum. In this sense the biased fixed prize raffle is the contest success function that can induce efficient public good provision with the lowest prize sum. A proof of this conjecture goes beyond the scope of this paper and is relegated to future research.

Risk-Neutrality and Subsidies. In our setup players have quasi-linear preferences which implies risk-neutrality. Relaxing this assumption might alter our results. Duncan (2002) considers risk-aversion in a setup with unbiased lotteries and demonstrates that, in contrast to Morgan (2000), higher prize values do not always lead to higher public good contributions. Under risk-aversion investments into lottery tickets become less attractive due to the stochastic nature of the lottery. A direct implication of this analysis is that depending on the specifics of the utility function there might exist a *maximal* prize value which induces an upper bound on public good provision (which will be second-best, i.e., below the optimal amount). For the case of biased lotteries with risk-neutral players, our previous analysis in Franke and Leininger (2014) estab-

[3]Concavity guarantees the existence of an equilibrium in pure strategies.

lished the existence of a *minimal* prize value which is required to induce the efficient amount of public good provision. Combining both insights implies that in some circumstances the maximal prize value (induced through risk-aversion) might actually be lower than the minimal prize value (required for the optimal bias to induce efficient public good provision), which would impede efficient public good provision under risk-aversion. These circumstances will be highly dependent on the degree of risk-aversion in combination with the degree of heterogeneity of the players; the analysis of this complex relation goes beyond the scope of this paper. Moreover, there exists a simple variation to circumvent these complications: Implementation of a weighted sharing rule instead of a biased lottery contest would transform the stochastic lottery into a subsidy scheme, where the subsidy depends in a proportional way on the relative but weighted individual contributions to the public good, comp. Remark 1 in Morgan (2000), without altering the structure of our model.[4] As this subsidy scheme does not involve any risk, violations of risk-neutrality would not have the mentioned implications and our analysis would apply without further modifications.

6 Concluding Remarks

We have considered biased fixed prize raffles for efficient public good provision. In these games contributions to the provision of a public good are elicited from consumers by offering a biased raffle with fixed prize R. The prize R is financed out of the proceeds from raffle ticket sales and only the surplus revenue beyond R goes towards provision of the public good. We have shown that the equilibrium outcome can always be generated by a *fair*; i.e. not biased, fixed prize raffle whose tickets are sold at different prices to different consumers; i.e. holding a biased fixed prize raffle can be understood as holding a fair raffle with a price-discriminating ticket seller. We then demonstrated that there is a close relation between the optimal degree of price-discrimination for lottery tickets and Lindahl pricing of the public good in the sense that optimal price-discrimination of lottery tickets is equivalent to setting Lindahl prices for the public good along with offering appropriate chances for winning the prize R.

[4] In Morgan (2000) the total subsidy amount is set-aside from total contributions; i.e., this modified scheme is still self-financing and satisfies budget-balance. This would not be true of a subsidy scheme that relies on additional resources to subsidize specific consumers, comp. for instance, Thomas and Wang (2017).

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